

blood-vessels, but prominently absent from the collecting lymphatic channels lying alongside of these vessels, thus supporting the hypothesis they formerly emitted, that these cells were merely links in a nutritive chain, not radicles of the lymphatics, even when, as in tendon, the cornea, &c., they are connected with the lymphatics. The paper is illustrated by about a dozen and a half of camera-lucida drawings of microscopical specimens in their possession.

II. "Refractive Indices of Glass." By J. HOPKINSON, D.Sc., M.A. Communicated by Professor G. G. STOKES, Sec. R.S. Received May 25, 1877.

Most of the following determinations were made two years ago. They were not published at once, because the results showed more variation than was expected. They are now made known for two reasons. First, most of the glasses examined are articles of commerce, and can be readily obtained by any person experimenting upon the physical properties of glass; these glasses only vary within narrow limits, and their variations may be approximately allowed for by a knowledge of their density. Second, most of the prisms having three angles from each of which determinations were made, the probable error of the mean is very small, and any error of the nature of a blunder is certainly detected.

The form in which to present these results was a matter of much consideration. A curve giving the refractive indices directly is unsuitable, for the errors of observation are less than the errors of curve-drawing would be. The theory of dispersion is not in a position to furnish a satisfactory rational formula. The most frequently used empirical formula is $\mu = a + b \frac{1}{\lambda^2} + c \frac{1}{\lambda^4} + \dots$, where μ is the wave-length of the ray to which μ refers. But to bring this within errors of observation it is necessary to include $\frac{1}{\lambda^6}$, which appears to be almost as important a term as $\frac{1}{\lambda^4}$. There are two points of importance in the selection of an empirical form: first, it must accurately represent the facts with the use of the fewest arbitrary parameters; second, it must be practically convenient for the purposes for which the results are useful.

In the present case the most convenient form is

$$\mu - 1 = a\{1 + bx(1 + cx)\},$$

where x is a numerical name for the definite ray of which μ is the refractive index. In the present paper line F, being intermediate between the strongest luminous and chemical rays, is taken as zero. Four glasses, Hard Crown, Soft Crown, Light Flint, and Dense Flint, are selected on account of the good accord of the results, and the mean of their refractive indices $\bar{\mu}$ is ascertained for each ray; this is taken as a standard scale in which $x = \bar{\mu} - \bar{\mu}_F$.

If f_0 be the focal distance of a compound lens for line F, f_0' , f_0'' , &c. of the component lenses, then

$$\frac{1}{f} = \sum \frac{1}{f_0^{(r)}} + \sum \frac{b^{(r)}}{f_0^{(r)}} \cdot v + \sum \frac{b^{(r)}c^{(r)}}{f_0^{(r)}} \cdot v^2,$$

f being the focal length for the ray denoted by v .

If there be two lenses in the combination

$$\frac{1}{f} = \frac{1}{f_0} + \frac{1}{f_0} \frac{b'b''}{b'' - b'} (c' - c'')v^2,$$

since the effect of changing the ray to be denoted by zero does not sensibly change the value of the coefficient $\frac{b'b''}{b'' - b'} (c' - c'')$, this may be taken as a measure of the irrationality of the combination.

Let there be three glasses (1), (2), (3); no combination free from secondary dispersion and of finite focal length can be made with these glasses if

$$\begin{vmatrix} 1, & 1, & 1 \\ b', & b'', & b''' \\ b'c', & b''c'', & b'''c''' \end{vmatrix} = 0.$$

Again, if the secondary chromatic aberration of (2) (1) is the same as that of (3) (1), then that of (2) (3) has also the same value, and the three glasses satisfy the above condition.

Prof. Stokes has expressed the character of glasses in the following manner:—Let a prism of small angle i be perfectly achromatized by two prisms of standard glasses with angles i' , i'' taken algebraically as regards sign, then

$$ai + a'i' + a''i'' = \text{deviation of any ray,}$$

$$abi + a'b'i' + a''b''i'' = 0,$$

$$abci + a'b'c'i' + a''b''c''i'' = 0;$$

hence

$$\frac{i'}{i''} = \frac{c' - c}{c - c'} \frac{a'b''}{a'b'}.$$

If $c = c''$ this ratio is zero, but if $c = c'$ it is infinite; let $\frac{i'}{i''} = \tan \phi$, then the angle ϕ may be taken with a and b as a complete specification of the optical properties of the glass. Prof. Stokes's method has a great advantage in the close correspondence between the values of i , i' , i'' and the powers of the component lenses of a perfectly achromatic object-glass, and also in the rapidity with which a determination can be made. The method adopted in this paper is convenient in the fact that a single standard glass is alone required.

The determinations were made with a spectrometer supplied to Messrs. Chance Bros. & Co. by Mr. Howard Grubb. The telescope and collimator are 2 inches aperture; the circle is 15 inches diameter, is graduated to $10'$, and reads by two verniers to $10''$.

The lines of the spectrum observed were generally A, B, C, D, E, b , F, (G), G, h , H_1 . D is the more refrangible of the pair of sodium lines, b

is the most refrangible of the group of magnesium lines, (G) is the hydrogen line near G.

The method of smoothing the results by the aid of each other has been, first, to calculate a , b , and c from the mean values of μ for the lines B, F, H₁; second, to calculate values of μ from the formula obtained; third, to plot on paper the differences between μ observed and μ calculated, and to draw a free-hand curve among the points, and then inversely to take μ for each line from the curve.

It appeared desirable to express the standard values of $\bar{\mu}$, which are the means of those for Hard Crown, Soft Crown, Light Flint, and Dense Flint, in terms of $\frac{1}{\lambda^2}$. In the following Table column

I. gives λ , the wave-length in 10^{-4} centims.;

II. the values of $\frac{1}{\lambda^2}$;

III. the standard values of $\bar{\mu}$;

IV. the values of $\bar{\mu}$ calculated from

$$\mu = a + b \frac{1}{\lambda^2} + c \frac{1}{\lambda^4}, \text{ where}$$

$$a = 1.539718,$$

$$b = 0.0056349,$$

$$c = 0.0001186;$$

V. the differences of III. and IV.;

VI. the values of $\bar{\mu}$ from the extended formula

$$\mu = a + b \frac{1}{\lambda^2} + c \frac{1}{\lambda^4} + d \frac{1}{\lambda^6}, \text{ where}$$

$$a = 1.538414,$$

$$b = 0.0067669,$$

$$c = -0.0001734,$$

$$d = 0.000023;$$

VII. The differences of III. and VI.

	I. λ .	II. $\frac{1}{\lambda^2}$.	III. $\bar{\mu}$.	IV.	V.	VI.	VII.
B.	.68668	2.12076	1.552201	1.552201	0.000000	1.552203	- 2
C.	.65618	2.32249	1.553491	1.553444	+0.000047	1.553481	+10
D.	.58890	2.88348	1.557030	1.556951	+0.000079	1.557033	- 3
E.	.52690	3.60200	1.561612	1.561553	+0.000059	1.561613	- 1
b.	.51667	3.74605	1.562530	1.562490	+0.000040	1.562538	- 8
F.	.48606	4.23272	1.565692	1.565692	0.000000	1.565693	- 1
G.	.43072	5.39026	1.573459	1.573536	-0.000077	1.573457	+ 2
h.	.41012	5.94536	1.577356	1.577409	-0.000053	1.577349	+ 7
H ₁ .	.39680	6.35121	1.580287	1.580287	0.000000	1.580289	- 2

It is interesting to remark that the curve representing μ in terms of $\frac{1}{\lambda^2}$ has a point of inflexion between C and D. An examination of the deviations from calculation for several glasses shows that probably all glasses exhibit a similar point of inflexion, the flints lower in the spectrum or in the ultra red, and the crowns nearer to the middle of the visible spectrum. This fact may be of importance in the theory of dispersion when a detailed theory becomes possible; at least it is important as showing how unsafe it would be to calculate μ for very long waves or ultra-violet waves from any formula of three terms.

The following Tables of results need little or no further explanation; the first line gives the refractive indices finally obtained and regarded as most probable, the second line gives the values of μ from the formula of three terms

$$\mu - 1 = a\{1 + bx(1 + cx)\},$$

and the last gives the mean of the actual observations.

Hard Crown.—Two specimens of this glass were examined.

α . A prism from the three angles of which determinations were obtained, density 2.48575.

$$a = 0.523145. \quad b = 1.3077. \quad c = -2.33.$$

	A.	B.	C.	D.	E.	b .
Most probable value of μ		1.513625	1.514568	1.517114	1.520331	1.520967
μ from formula.....		1.513624	1.514560	1.517099	1.520327	1.520966
Mean of observed values	1.511755	1.513624	1.514571	1.517116	1.520324	1.520962

	F.	(G.)	G.	h .	H ₁ .
Most probable value of μ	1.523139	1.527994	1.528353	1.530902	1.532792
μ from formula.....	1.523145	1.528003	1.528362	1.530906	1.532789
Mean of observed values	1.523145	1.527996	1.528348	1.530904	1.532789

β . A prism with two angles of about 45° , density= 2.48664 .

The first line gives the mean observed indices, and the second the differences from the most probable values of α .

	A.	B.	C.	D.	E.	b.
Mean of observed values	1.572296	1.514156	1.515105	1.517623	1.520860	1.521474
	(0.000541)	0.000531	0.000537	0.000509	0.000529	0.000507
	F.	(G.)	G.	h.	H ₁ .	
Mean of observed values	1.523657	1.528533	1.528856	1.531439	1.533318	
	0.000518	0.000539	0.000503	0.000537	0.000526	

Soft Crown.—The prism has three angles from which the mean is taken, density= 2.55035 .

$$a=0.5209904, \quad b=1.4034, \quad c=-1.58.$$

	A.	B.	C.	D.	E.	b.
Most probable value of μ		1.510916	1.511904	1.514591	1.518010	1.518686
μ from formula.....		1.510918	1.511900	1.514574	1.517991	1.518670
Mean of observed values	1.508956	1.510918	1.511910	1.514580	1.518017	1.518678
	F.	(G.)	G.	h.	H ₁ .	
Most probable value of μ	1.520996	1.526207	1.526595	1.529359	1.531416	
μ from formula.....	1.520994	1.526215	1.526603	1.529364	1.531418	
Mean of observed values	1.520994	1.526208	1.526592	1.529360	1.531415	

Titano-silicic Crown.—This glass was made on the suggestion of Professor Stokes in the hope of obtaining a glass of good quality which

should be perfectly achromatic with a flint. The determinations were made on two angles; density=2·55255.

$$a=0\cdot550466, \quad b=1\cdot5044, \quad c=-0\cdot93.$$

	B.	C.	D.	E.	b.	F.
Most probable value of μ	1·539155	1·540255	1·543249	1·547088	1·547852	1·550471
μ from formula.....	1·539152	1·540249	1·543236	1·547074	1·547839	1·550466
Mean of observed values	1·539157	1·540253	1·543248	1·547088	1·547852	1·550471

	(G.)	G.	h.	H ₁ .		
Most probable value of μ	1·556386	1·556830	1·559999	1·562392		
μ from formula.....	1·556407	1·556852	1·560015	1·562388		
Mean of observed values	1·556394	1·556825	1·559998	1·562392		

Extra Light Flint Glass.—Determinations made from three angles, density=2·86636.

$$a=0\cdot549123, \quad b=1\cdot7064, \quad c=-0\cdot198.$$

	A.	B.	C.	D.	E.	b.
Most probable value of μ		1·536450	1·537673	1·541011	1·545306	1·546166
μ from formula.....		1·536448	1·537663	1·540993	1·545296	1·546158
Mean of observed values	1·534067	1·536450	1·537682	1·541022	1·545295	1·546169

	F.	(G.)	G.	h.	H ₁ .	
Most probable value of μ	1·549121	1·555863	1·556372	1·560010	1·562760	
μ from formula.....	1·549123	1·555881	1·556390	1·560026	1·562760	
Mean of observed values	1·549125	1·555870	1·556375	1·559992	1·562760	

Light Flint Glass.—Determinations made from three angles,
density=3·20609.

$$a=0\cdot583887. \quad b=1\cdot9605. \quad c=+0\cdot53.$$

	B.	C.	D.	E.	<i>b.</i>	F.
Most probable value of μ	1·568558	1·570011	1·574015	1·579223	1·580271	1·583886
μ from formula.....	1·568553	1·570009	1·574017	1·579226	1·580274	1·583887
Mean of observed values	1·568558	1·570007	1·574013	1·579227	1·580273	1·583881
	(G.)	G.	<i>h.</i>	H ₁ .		
Most probable value of μ	1·592190	1·592824	1·597332	1·600727		
μ from formula.....	1·592185	1·592815	1·597322	1·600724		
Mean of observed values	1·592184	1·592825	1·597332	1·600717		

Dense Flint.—Determinations from three angles, density=3·65865.

$$a=0\cdot634744. \quad b=2\cdot2694. \quad e=1\cdot48.$$

	B.	C.	D.	E.	<i>b.</i>	F.
Most probable value of μ	1·615701	1·617484	1·622414	1·628895	1·630204	1·634748
μ from formula.....	1·615700	1·617488	1·622427	1·628904	1·630210	1·634744
Mean of observations ...	1·615704	1·617477	1·622411	1·628882	1·630208	1·634748
	(G.)	G.	<i>h.</i>	H ₁ .		
Most probable value of μ	1·645267	1·646068	1·651840	1·656219		
μ from formula.....	1·645258	1·646060	1·651837	1·656222		
Mean of observations ...	1·645268	1·646071	1·651830	1·656229		

Extra Dense Flint.—Determinations from only one angle,
density=3·88947.

$$a=0\cdot664226. \quad b=2\cdot4446. \quad c=1\cdot87.$$

	A.	B.	C.	D.	E.	b.
Most probable value of μ and μ from formula ...		1·642874	1·644866	1·650388	1·657653	1·659122
Observed	1·639143	1·642894	1·644871	1·650374	1·657631	1·659108

	F.	(G.)	G.	h.	H ₁ .	
Most probable value of μ and μ from formula }	1·664226	1·676111	1·677019	1·683577	1·688569	
Observed	1·664246	1·676090	1·677020	1·683575	1·688590	

Double Extra Dense Flint.—Determinations from two angles,
density=4·42162.

$$a=0\cdot727237. \quad b=2\cdot7690. \quad c=2\cdot70.$$

	A.	B.	C.	D.	E.	b.
Most probable value of μ and μ from formula }		1·701060	1·703478	1·710201	1·719114	1·720924
Mean of observations ...	1·696531	1·701080	1·703485	1·710224	1·719081	1·720908

	F.	(G.)	G.	h.	H ₁ .	
Most probable value of μ and μ from formula }	1·727237	1·742063	1·743204	1·751464	1·757785	
Mean of observations ...	1·727257	1·742058	1·743210	1·751485		

The following Table gives the value of $\frac{bb'}{b'-b}(c-c')$ for each glass when combined with the standard. It serves to show how little there is to choose between the glasses ordinarily used.

Hard Crown.	Soft Crown.	Titanic Crown.	Extra Light Flint.	Light Flint.	Dense Flint.	Extra Dense Flint.	Double Extra Dense Flint.
-11·7	-10·7	-9·4	-9·9	-9·4	-11·8	-11·9	-13·2